THERMO-MECHANICAL PRESSURE ANALYSIS FOR HVDC CABLES

Ziyi **HUANG**, James A **PILGRIM;** University of Southampton, (UK),
zh2g09@soton.ac.uk, jp2@ecs.soton.ac. uk

ABSTRACT

For some mass-impregnated (MI) paper insulated HVDC cables, unacceptably high pressures or pressure drops occur at the insulation-sheath interface under rapid loading changes, due to a strong impregnant thermal expansion or contraction. As this can degrade cables by either causing sheath plastic deformations or introducing voids into the insulation, it is important to investigate/model this thermo-mechanical phenomenon. Based on the theory of elasticity, this paper presents an analytical calculation of interfacial pressures as a function of the cable loading. Results obtained will benefit both cable operators and manufactures by provide guidelines of rating calculations and insulation designs, following a thermo-mechanical constraint.

KEYWORDS

Current rating, Elasticity, Interfacial stress, Mass impregnated paper, Thermal expansion

NOMENCLATURE

E	Young's modulus (N.m^{-2})
E_c	Young's modulus of conductor (N.m^{-2})
E_i	Young's modulus of insulation (N.m^{-2})
E_s	Young's modulus of sheath (N.m^{-2})
I	cable loading (A)
I_{t-m}	Thermo-mechanical rating (A)
L	Object initial length (m)
ΔL	Change of length due to thermal expansion (m)
P_i	Inner pressure on the annulus inner radius (N.m^{-2})
P_o	Outer pressure on the annulus outer radius (N.m^{-2})
P_1	Conductor/insulation interfacial pressure (N.m^{-2})
P_2	Insulation/sheath interfacial pressure (N.m^{-2})
P_3	External pressure on sheath outer radius (N.m^{-2})
r_{ai}	Annulus inner radius (m)
r_{ao}	Annulus outer radius (m)
r_c	Conductor outer radius (m)
r_i	Insulation outer radius (m)
r_i	Sheath outer radius (m)
R_{dc}	Dc resistance of cable conductor (Ω.m^{-1})
T_1	Thermal resistance of the insulation (K.m.W^{-1})
T_2	Sheath/armour Thermal resistance (K.m.W^{-1})
T_3	Thermal resistance of serving (K.m.W^{-1})
T_4	Thermal resistance of backfill (K.m.W^{-1})
T_{total}	Total thermal resistance from annulus outer boundary to ambient (K.m.W^{-1})
u	Radial displacement (m)
u_c	Conductor radial displacement (m)
u_i	Insulation radial displacement (m)
u_s	Sheath radial displacement (m)
v	Poisson's ratio
v_c	Poisson's ratio of conductor
v_i	Poisson's ratio of insulation
v_s	Poisson's ratio of sheath
V	Object initial volume (m^3)
ΔV_t	Volume change by thermal expansion (m^3)
ΔV_m	Volume change by mechanical compression (m^3)
ρ	Thermal resistivity (K.m.W^{-1})
ρ_c	Thermal resistivity of conductor (K.m.W^{-1})
ρ_i	Thermal resistivity of insulation (K.m.W^{-1})
ρ_s	Thermal resistivity of sheath (K.m.W^{-1})
θ	Temperature (^{0}C)
θ_{amb}	Ambient temperature remote from cable (^{0}C)
$\Delta\theta$	Change in temperature (^{0}C)
σ_r	Radial stress (N.m^{-2})
σ_y	Maximum yield shear stress (N.m^{-2})
σ_θ	Circumferential stress (N.m^{-2})
ϕ	Porosity of Kraft paper
β_{ip}	Compressibility of the Kraft paper (m^3.N^{-1})
β_{io}	Compressibility of the impregnant (m^3.N^{-1})
α_c	Conductor linear thermal expansion coefficient (1.K^{-1})
α_i	Insulation linear thermal expansion coefficient (1.K^{-1})
α_s	Sheath linear thermal expansion coefficient (1.K^{-1})
α_{io}	Impregnant (oil) volumetric thermal expansion coefficient (1.K^{-1})
α_{ip}	Kraft paper volumetric thermal expansion coefficient (1.K^{-1})
α_L	Linear thermal expansion coefficient (1.K^{-1})
α_v	Volumetric thermal expansion coefficient (1.K^{-1})
$\alpha_{v\text{-}paper}$	Volumetric thermal expansion coefficient of Kraft paper (1.K^{-1})
$\alpha_{v\text{-}impregnant}$	Volumetric thermal expansion coefficient of dielectric impregnant (1.K^{-1})

INTRODUCTION

For some subsea HVDC cables with mass-impregnated (MI) paper insulation, it may be necessary to impose restrictions on the cable operation under low ambient temperatures. It is believed that unacceptably high interfacial pressure transients, due to the strong thermal expansion of the high viscosity impregnant, can occur under rapid loading increases, causing plastic deformation of cable sheath. Moreover, big pressure drops can also occur during fast cooling, due to a higher contraction rate of the dielectric impregnant than that of the metallic sheath. Consequently, the overall dielectric strength of the cable is reduced.

Although previous work has modeled the pressure effects for MI-type paper insulated dc cables [1], it adopts a numerical procedure to model the outwards impregnant movement due to thermal expansion. This work develops an analytical methodology for the interfacial pressure calculation based on the classic elasticity theory. The main challenge is to properly model the insulation layer, which is a mixture of Kraft paper and high viscosity impregnant (mineral oil T2015). To achieve that, two extreme situations are considered. At 'low temperature', the insulation is assumed to be isotropic and elastic with equivalent thermal and mechanical properties because the impregnant viscosity remains high. However at 'high temperature', an insulation consisting of 'rigid' paper and compressible impregnant liquid is assumed with uniform stress distribution. This assumption depends on the fact that the impregnant oil has a much higher thermal

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expansion coefficient than Kraft paper ($\alpha_{v\text{-}impregnant} \approx 7\text{x}10^{-4}$ K^{-1}, $\alpha_{v\text{-}paper} \approx 4\text{x}10^{-5}$ K^{-1} [2]) and its viscosity decreases with increasing temperature.

This paper firstly introduces several standard thermal and mechanical formulas which contribute to the analytical methodology development. Secondly, project approach and justification is presented, followed by detailed derivations of the cable pressure as a function of cable loading. Thirdly, a concept of thermo-mechanical rating is introduced with a sample calculation. Finally, summary and proposed future work is outlined.

Linear and volumetric thermal expansion

For most engineering materials in either solid or liquid state, thermal expansion is the tendency of matter to change in volume in response to a change in temperature [3], which is formulated as:

$$\Delta L = \alpha_L L \Delta\theta \tag{1}$$

$$\Delta V = \alpha_V V \Delta\theta \tag{2}$$

Note that for isotropic materials, $\alpha_V = 3\alpha_L$.

Temperature distribution within annulus

Within HVDC cables, conductor joule loss and dielectric leakage current loss are two main heat sources, however the latter one is normally negligible compared to the joule loss ($W_{joule} \approx 20 - 30$ W.m^{-1}, $W_{leakage} < 1$ W.m^{-1}). By modelling the two-dimensional dc cable cross section as concentric annuli, the temperature distribution within each annular layer is:

$$\theta(r) = I^2 R_{dc}\left[\frac{\rho}{2\pi}\ln\left(\frac{r_{ao}}{r}\right) + T_{total}\right] + \theta_{amb} \tag{3}$$

Thus by assuming the cable and ambient have the same temperature before load is applied, the temperature rise, $\Delta\theta$, becomes:

$$\Delta\theta(r) = \theta(r) - \theta_{amb} = I^2 R_{dc}\left[\frac{\rho}{2\pi}\ln\left(\frac{r_{ao}}{r}\right) + T_{total}\right] \tag{4}$$

Plain stress and plain strain

In the theory of elasticity, plain stress and plain strain are two distinct simplified models for 2D plane analyses [4].

The plain stress is defined to be a stress state where the normal stress and associated shear stresses (z direction), directed perpendicular to the x-y plane, are assumed to be zero. This model applies to practical situations where objects have one dimension extremely small compared to the other two or extremely long cables without end constraints (free longitudinal expansion at cable ends). Particularly for an annulus under thermal expansion (with inner/outer radiuses r_i/r_o subject to inner/outer compressive pressures P_i/P_o), the resulting radial displacement and two principle stresses are formulated by:

$$u(r) = (1+v)\frac{\alpha_L}{r}\int_a^r \Delta\theta(r) r dr + K_1 r + \frac{K_2}{r} \tag{5}$$

$$\sigma_r(r) = \frac{-\alpha_L E}{r^2}\int_a^r \Delta\theta(r) r dr + \frac{E}{1-v^2}\left[K_1(1+v) - \frac{K_2(1-v)}{r^2}\right] \tag{6}$$

$$\sigma_\theta(r) = \frac{\alpha_L E}{r^2}\int_a^r \Delta\theta(r) r dr - \alpha_L E \Delta\theta(r) + \frac{E}{1-v^2}\left[K_1(1+v) + \frac{K_2(1-v)}{r^2}\right] \tag{7}$$

Where; constants K_1 and K_2 are defined by boundary conditions. For normal annuli, $a = r_{ai}$ and for solid discs, $a = K_2 = 0$.

Conversely, the plain strain is defined to be another stress state where the normal strain and associated shear strains (z direction), directed perpendicular to the x-y plane, are assumed to be zero. This model applies to practical situations where objects have one dimension extremely bigger compared with the other two or extremely long cables with constrained ends. Therefore for annuli under boundary conditions the same as previously:

$$u(r) = \frac{(1+v)\alpha_L}{(1-v)r}\int_a^r \Delta\theta(r) r dr + K_1 r + \frac{K_2}{r} \tag{8}$$

$$\sigma_r(r) = \frac{-\alpha_L E}{(1-v)r^2}\int_a^r \Delta\theta(r) r dr + \frac{E}{1+v}\left[\frac{K_1}{1-2v} - \frac{K_2}{r^2}\right] \tag{9}$$

$$\sigma_\theta(r) = \frac{\alpha_L E}{(1-v)r^2}\int_a^r \Delta\theta(r) r dr - \frac{\alpha_L E \Delta\theta(r)}{1-v} + \frac{E}{1+v}\left[\frac{K_1}{1-2v} + \frac{K_2}{r^2}\right] \tag{10}$$

Where; constants K_1 and K_2 are defined by the boundary conditions. For normal annuli, $a = r_{ai}$ and for solid cylinders, $a = K_2 = 0$.

Generally, the radial stress, σ_r, and circumferential stress, σ_θ, are defined, by default, as two principle stresses perpendicular to each other.

Mechanical failure/yield

A well recognized failure/yield criterion applicable to cable sheath is Tresca's (maximum shear stress) criterion for ductile materials [5], such as aluminium and lead.

Under Tresca's criterion, yield is caused by the slippage of crystal planes along the maximum shear stress surface. Therefore, the criterion requires the maximum shear stress (principle stress difference) to be less than the material yield shear stress, which can be obtained from a uniaxial tensile test, calculated by

$$|\sigma_\theta - \sigma_r| \le \sigma_y \tag{11}$$

This gives the limiting criteria as the elastic deformation limit, above which plastic deformation occurs. Within a plain stress analysis, substituting (4), (6) and (7) into (11) gives:

$$\left|\frac{1}{r^2}\left\{\alpha_L E I^2 R_{dc} r_{ai}^2\left[\frac{\rho}{2\pi}\ln\left(\frac{r_{ai}}{r_{ao}}\right) - \frac{\rho}{4\pi} - T_{total}\right] + \frac{2EK_2}{1+v}\right\} + \frac{\alpha_L E I^2 R_{dc}\rho}{4\pi}\right| \le \sigma_y \tag{12}$$

From (12), the maximum sheath stress is found to be at the annulus inner radius where $r = r_{ai}$. Moreover, same conclusion is found for a plain strain analysis by substituting (4), (9) and (10) into (11), which is shown as:

$$\left|\frac{1}{r^2}\left\{\frac{\alpha_L E I^2 R_{dc} r_{ai}^2}{1-v}\left[\frac{\rho}{2\pi}\ln\left(\frac{r_{ai}}{r_{ao}}\right) - \frac{\rho}{4\pi} - T_{total}\right] + \frac{2EK_2}{1+v}\right\} + \frac{\alpha_L E I^2 R_{dc}\rho}{4\pi(1-v)}\right| \le \sigma_y \tag{13}$$

TECHNICAL APPROACH

To calculate the cable internal pressure as a function of current loading, a multi-physics combination of elasticity theory, thermodynamics and electrical fundamentals is required. Therefore, Table 1 below summaries the necessary knowledge input from each of the above three physical theories, and Fig. 1 shows the overall approach.

Theory of elasticity	· Plain stress/strain analysis for cylindrical coordinates · Mechanical failure criterion and its location
Thermodynamics	· Linear/volumetric thermal expansion · Temperature distribution for cylindrical coordinates
Electrical fundamentals	· Ohmic loss calculation

Table 1: Summary of conceptual input

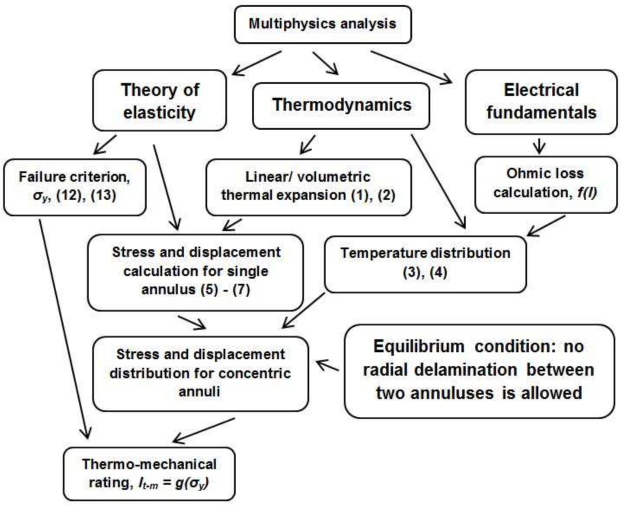

Fig. 1: Approach logic

THERMO-MECHANICAL PRESSURE ANALYSIS

In this section, the full analytical derivation of the interfacial pressure as a function of cable loading, I, and external controllable pressure, P_3, are presented for both plain stress and plain strain analyses. The model consists of three concentric layers representing cable conductor (outer radius r_c), insulation (outer radius r_i) and sheath (outer radius r_s). P_1 is the absolute value of the interfacial pressure between conductor and insulation and P_2 is that between insulation and sheath. By default, each layer is subjected to compressive pressures (with negative sign) on both inner and outer surfaces.

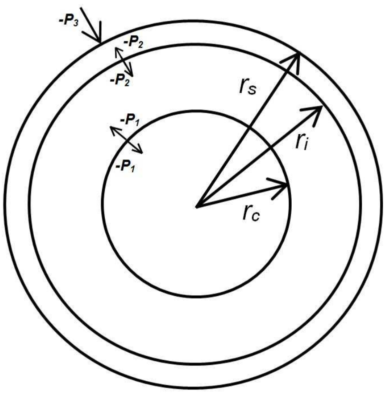

Fig. 2: Three layer cable cross section

Notes that the calculation of variable constants are summarised in the Appendix.

Plain stress analysis

'Low temperature' assumption

Under the 'low temperature' assumption, the insulation is assumed to be isotropic and elastic with equivalent mechanical and thermal parameters.

For the cable conductor subject to thermal expansion and external pressure, $-P_1$, at $r = r_c$, from (4) and (5):

$$u_c(r_c) = C_1 I^2 + D_1 P_1 \tag{14}$$

For the cable insulation layer under thermal expansion and internal/ external compressive pressures, $-P_1$/ $-P_2$, at r_c/ r_i, substituting (4) into (5) gives:

$$u_i(r_i) = C_5 I^2 + D_4 P_1 + F_3 P_2 \tag{15}$$

$$u_i(r_c) = C_3 I^2 + D_3 P_1 + F_2 P_2 \tag{16}$$

For the cable sheath layer under thermal expansion and internal/ external compressive pressures $-P_2$/ $-P_3$ at r_i/ r_s:

$$u_s(r_i) = C_7 I^2 + F_5 P_2 + G_2 P_3 \tag{17}$$

As radial delamination is not allowed, the radial displacements on both sides of an interface should compensate each other. Therefore, the conditions $u_c(r_c) = u_i(r_c)$ and $u_i(r_i) = u_s(r_i)$ lead to the following simultaneous equations:

$$\begin{cases} (C_3 - C_1)I^2 + (D_3 - D_1)P_1 + F_2 P_2 = 0 & (18) \\ (C_7 - C_5)I^2 + (F_5 - F_3)P_2 + G_2 P_3 - D_4 P_1 = 0 & (19) \end{cases}$$

Eliminating either P_1 or P_2 from (18) and (19) gives:

$$P_1 = f_1(I, P_3) = C_8 I^2 + G_3 P_3 \tag{20}$$

$$P_2 = f_2(I, P_3) = C_9 I^2 + G_4 P_3 \tag{21}$$

Finally, applying boundary conditions $\sigma_r(r_c) = -P_1 = -f_1(I, P_3)$, $\sigma_r(r_i) = -P_2 = -f_2(I, P_3)$ and $\sigma_r(r_s) = -P_3$ into (6) and (7), both radial and circumferential stresses, σ_r and σ_θ, can be determined by providing cable loading, I, and external controllable pressure, P_3.

'High temperature' assumption

Under the 'high temperature' assumption, the insulation consists of Kraft paper and impregnant with distinct compressibility (bulk modulus) and thermal expansion coefficient. As the impregnant oil has a much greater thermal expansion and its viscosity drops with an increasing temperature, uniform pressure distribution is assumed across the insulation layer.

Denote ϕ the Kraft paper porosity, the total volumetric expansion for a unit cable length is:

$$\Delta V_t = \phi \int_a^b \alpha_{io} \Delta\theta(r) 2\pi r dr + (1-\phi) \int_a^b \alpha_{ip} \Delta\theta(r) 2\pi r dr \tag{22}$$

By substituting (4) into (23), the volumetric thermal expansion of the insulation becomes:

$$\Delta V_t = H_1 I^2 \tag{23}$$

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Assuming the dielectric inner pressure has magnitude of P_1, the compression of the insulation layer is:

$$\Delta V_m = -\pi(r_i^2 - r_c^2)\left[\phi\beta_{io} + (1-\phi)\beta_{ip}\right]P_1 = J_4 P_1 \quad (24)$$

For the radial displacements of conductor outer boundary and sheath inner boundary:

$$u_c(r_c) = H_4 I^2 + J_3 P_1 \quad (25)$$

$$u_s(r_i) = H_3 I^2 + J_2 P_1 + L_2 P_3 \quad (26)$$

Based on the volumetric equilibrium, the dielectric thermal expansion is to be compensated by its own mechanical compression and the volumetric variation of both cable conductor and sheath, which leads to equation (27):

$$\Delta V_t + \Delta V_m = \left\{\pi r_c^2 - \pi(r_c + u_c(r_c))^2\right\} + \left\{\pi(r_i + u_s(r_i))^2 - \pi r_i^2\right\} \quad (27)$$

Therefore by substituting equations (23) to (26) into (27), we obtain a quadratic equation of P_1 with solution as:

$$H_5 P_1^2 + J_5 P_1 + L_3 = 0 \quad (28)$$

$$P_1 = f_3(I, P_3) = \frac{-J_5 + \sqrt{J_5^2 - 4H_5 L_3}}{2H_5} \quad (29)$$

Finally, applying boundary conditions $\sigma_r(r_i) = -P_1 = -f_3(I, P_3)$ and $\sigma_r(r_s) = -P_3$ into (6) and (7), both radial and circumferential stresses, σ_r and σ_θ, can be determined by providing cable loading, I, and external controllable pressure, P_3.

Plain strain analysis

Within the plain strain analysis, the same analytical derivation applies, except for replacing (5), (6) and (7) with (8), (9) and (10). Therefore only final equations are outlined in this section.

Under the 'low temperature' assumption,

$$P_1 = g_1(I, P_3) = M_8 I^2 + N_3 P_3 \quad (30)$$

$$P_2 = g_2(I, P_3) = M_9 I^2 + N_4 P_3 \quad (31)$$

By applying boundary conditions $\sigma_r(r_c) = -P_1 = -f_4(I, P_3)$, $\sigma_r(r_i) = -P_2 = -f_5(I, P_3)$ and $\sigma_r(r_s) = -P_3$ into (8) and (9), both radial and circumferential stresses, σ_r and σ_θ, can be determined by providing cable loading, I, and external controllable pressure, P_3.

Under the 'high temperature' assumption, calculate the insulation volumetric thermal expansion using (22) and boundary radial displacements through (8):

$$\Delta V_t = S_1 I^2 \quad (32)$$

$$\Delta V_m = W_1 P_1 \quad (33)$$

$$u_c(r_c) = S_4 I^2 + W_4 P_1 \quad (34)$$

$$u_s(r_i) = S_3 I^2 + W_3 P_1 + X_2 P_3 \quad (35)$$

Based on the volumetric equilibrium, substituting equations (32) to (35) into (27), P_1 can be expressed as a function of cable loading, I, and external controllable pressure, P_3. Finally, with boundary conditions, $\sigma_r(r_i) = -P_1 = -f_6(I, P_3)$ and $\sigma_r(r_s) = -P_3$, both radial and circumferential stresses, σ_r (8) and σ_θ (9), can be determined by providing cable loading, I, and external controllable pressure, P_3.

THERMO-MECHANICAL RATING METHODOLOGY

One important application of the above cable pressure calculation is to help develop the so-called 'thermo-mechanical rating' methodology. Different from the well-known thermal rating in IEC60287 [6] (limited by maximum conductor temperature) and thermoelectric rating [7] (limited by maximum dielectric field stress), it follows the cable sheath yield criterion, which prevents any potential plastic deformations.

In this chapter, the methodology is only derived for a 'high temperature' plain stress analysis, following Tresca's criterion. However, it can be transferred to a plain strain analysis through the same procedure if necessary. Subsequently, a sample calculation with practical cable parameters is presented.

By applying P_1 in (29) and P_3 to (6) as boundary conditions, K_2 for the cable sheath layer is given by:

$$K_2 = \frac{r_i^2 r_s^2 (F_4 P_1 - C_6 I_{t-m}^2 - G_1 P_3)}{(1 - v_s)(r_i^2 - r_s^2)} \quad (36)$$

Substitute (36) into (12) and rearrange the equation for cable loading, I, at the sheath inner radius. The thermo-mechanical rating is:

$$I_{t-m} = \sqrt{\frac{(1+v_s) r_i^2 \sigma_y - 2E_s K_2}{(1+v_s) r_i^2 \alpha_s E_s R_{dc}\left[\frac{\rho_s}{2\pi}\ln\left(\frac{r_i}{r_s}\right) - T_3 - T_4\right]}} \quad (37)$$

Note that as I_{t-m} appears on both sides of (37), through the dependency in K_2, an iterative solution is required.

In the following practical example, the key cable parameters [8] are summarized in Table 2.

Parameter	Value	Parameter	Value
E_c	1.2e11	E_s	6.9e10
v_c	0.35	v_s	0.34
T_1	0.5081	T_2	0
T_3	0.0434	T_4	0.909
σ_y	1.3e8	β_{io}	10e-9 - 10e-11
α_c	1.7e-5	α_{io}	7e-4
α_s	2.35e-5	ρ_c	2.5e-3
ρ_i	6	ρ_s	4.1e-3
r_c	30e-3	r_i	51.5e-3
r_s	55e-3	R_{dc}	7.71e-6
ϕ	0.2 - 0.5	P_3	1e6 (100m depth)

Table 2: Practical cable parameter list

Due to the constraints on available dielectric property data, the following assumptions are made:

- Compared to the dielectric oil in semi-liquid state, kraft paper is assumed 'rigid' with negligible thermal expansion and mechanical compression.
- The mineral oil compressibility, β_{io}, varies due to different composites.
- The highly porous kraft paper is considered to have porosity between 0.2 to 0.5.
- The volume of oil is sufficient to avoid air gaps in the undeformed sheath

Based on the developed methodology, constants are calculated in Table 3 and a *σ-I* relations under various paper porosities is shown in Fig. 3.

β_{io} = 1.8e-9, ϕ = 0.5 and $I_{t\text{-}m}$ = 2275			
C_6	1.06e-11	F_4	-1.28e-11
G_1	-1.28e-11	H_1	1.71e-11
H_2	1.06e-11	H_3	8.89e-12
H_4	5.79e-12	H_5	1.05e-22
J_1	-1.28e-11	J_2	1.03e-11
J_3	-1.64e-13	J_4	-4.91e-12
J_5	2.63e-12	L_1	-1.28e-11
L_2	-1.07e-11	L_3	-2.64e-5

Table 3: Sample variable coefficient calculations

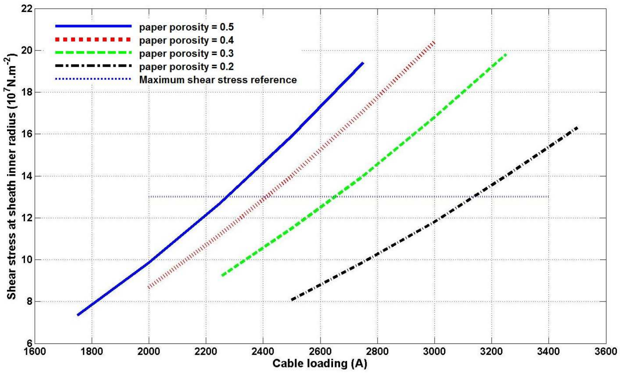

Fig. 3: Sheath shear stress plot with various loadings

In Fig. 3, the thermo-mechanical rating (intersection with reference line) ranges from 2200A to 3200A, and it increases with decreasing paper porosity. This is expected as the total volume of oil has decreased for lower porosity papers, hence reducing the extent of the dielectric expansion.

SUMMARY AND FUTURE WORK

An analytical methodology of calculating cable internal pressure has been outlined, which is based on the theory of elasticity. Further, the concept of thermo-mechanical rating is introduced, calculating the cable rating as a function of sheath shear stress allowance, controllable external pressure and insulation properties. As it is an initial research approach, this work adopts two extreme assumptions ('high' or 'low' temperature), which refers to the upper/lower boundaries of available ratings. Further work is required to simplify the equation set to reduce the complexity of the implementation. An additional difficult is the measurement of the insulation thermal and mechanical properties, some of which are not well known at present.

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APPENDIX

This appendix summarises the calculation of variable parameters within the analytical derivation.

$$C_{10} = \frac{-r_i^2 r_s^2 C_6}{(1-v_s)(r_i^2 - r_s^2)} \qquad \text{(A.1)}$$

$$C_9 = \frac{D_4(C_3 - C_1) + (D_3 - D_1)(C_7 - C_5)}{D_4 F_2 + (D_3 - D_1)(F_5 - F_3)} \qquad \text{(A.2)}$$

$$C_8 = \frac{F_2(C_7 - C_5) - (F_5 - F_3)(C_3 - C_1)}{(F_5 - F_3)(D_3 - D_1) + F_2 D_4} \qquad \text{(A.3)}$$

$$C_7 = \frac{r_i r_s^2 C_6}{(1+v_s)(r_s^2 - r_i^2)} - \frac{r_i r_s^2 C_6}{(1-v_s)(r_i^2 - r_s^2)} \qquad \text{(A.4)}$$

$$C_6 = (1-v_s^2)\alpha_s R_{dc}\left\{\frac{\rho_s}{2\pi}\left[\frac{r_i^2 \ln(r_i)}{2r_s^2} - \frac{r_i^2}{4r_s^2} - \frac{\ln(r_s)}{2} + \frac{1}{4}\right] + \frac{r_s^2 - r_i^2}{2r_s^2}\left[\frac{\rho_s \ln(r_s)}{2\pi} + T_3 + T_4\right]\right\} \qquad \text{(A.5)}$$

$$C_5 = C_4 + \frac{r_i^3 C_2}{(1+v_i)(r_i^2 - r_c^2)} - \frac{r_i r_c^2 C_2}{(1-v_i)(r_c^2 - r_i^2)} \qquad \text{(A.6)}$$

$$C_4 = \frac{(1+v_i)\alpha_i R_{dc}}{2}\left\{\frac{\rho_i}{\pi}\left[\frac{r_c^2 \ln(r_c)}{2r_i} - \frac{r_c^2}{4r_i} - \frac{r_i \ln(r_i)}{2} + \frac{r_i}{4}\right] + \frac{r_i^2 - r_c^2}{r_i}\left[\frac{\rho_i \ln(r_i)}{2\pi} + T_2 + T_3 + T_4\right]\right\} \qquad \text{(A.7)}$$

$$C_3 = \frac{r_c r_i^2 C_2}{(1+v_i)(r_i^2 - r_c^2)} - \frac{r_c r_i^2 C_2}{(1-v_i)(r_c^2 - r_i^2)} \qquad \text{(A.8)}$$

$$C_2 = (1-v_i^2)\alpha_i R_{dc}\left\{\frac{\rho_i}{2\pi}\left[\frac{r_c^2 \ln(r_c)}{2r_i^2} - \frac{r_c^2}{4r_i^2} - \frac{\ln(r_i)}{2} + \frac{1}{4}\right] + \frac{r_i^2 - r_c^2}{2r_i^2}\left[\frac{\rho_i \ln(r_i)}{2\pi} + T_2 + T_3 + T_4\right]\right\} \qquad \text{(A.9)}$$

$$C_1 = R_{dc}\alpha_c r_c(T_1 + T_2 + T_3 + T_4) \qquad \text{(A.10)}$$

$$D_4 = \frac{-r_c^2 r_i D_2}{(1+v_i)(r_i^2 - r_c^2)} + \frac{r_i r_c^2 D_2}{(1-v_i)(r_c^2 - r_i^2)} \qquad \text{(A.11)}$$

$$D_3 = \frac{-r_c^3 D_2}{(1+v_i)(r_i^2 - r_c^2)} + \frac{r_c r_i^2 D_2}{(1-v_i)(r_c^2 - r_i^2)} \qquad \text{(A.12)}$$

$$D_2 = -(1-v_i^2)/E_i \qquad \text{(A.13)}$$

$$D_1 = -r_c(1-v_c)/E_c \qquad \text{(A.14)}$$

$$F_6 = \frac{r_i^2 r_s^2 F_4}{(1-v_s)(r_i^2 - r_s^2)} \qquad \text{(A.15)}$$

$$F_5 = \frac{-r_i^3 F_4}{(1+v_s)(r_s^2 - r_i^2)} + \frac{r_i r_s^2 F_4}{(1-v_s)(r_i^2 - r_s^2)} \qquad \text{(A.16)}$$

Close and Return

$$F_4 = -(1 - v_s^2) / E_s \tag{A.17}$$

$$F_3 = \frac{r_i^3 F_1}{(1+v_i)(r_i^2 - r_c^2)} - \frac{r_i r_c^2 F_1}{(1-v_i)(r_c^2 - r_i^2)} \tag{A.18}$$

$$F_2 = \frac{r_c r_i^2 F_1}{(1+v_i)(r_i^2 - r_c^2)} - \frac{r_c r_i^2 F_1}{(1-v_i)(r_c^2 - r_i^2)} \tag{A.19}$$

$$F_1 = -(1 - v_i^2) / E_i \tag{A.20}$$

$$C_5 = \frac{-r_i^2 r_s^2 G_1}{(1-v_s)(r_i^2 - r_s^2)} \tag{A.21}$$

$$G_4 = \frac{G_2(D_1 - D_3)}{D_4 F_2 + (D_3 - D_1)(F_5 - F_3)} \tag{A.22}$$

$$G_3 = \frac{F_2 G_2}{(F_5 - F_3)(D_3 - D_1) + F_2 D_4} \tag{A.23}$$

$$G_2 = \frac{r_i r_s^2 G_1}{(1+v_s)(r_s^2 - r_i^2)} - \frac{r_i r_s^2 G_1}{(1-v_s)(r_i^2 - r_s^2)} \tag{A.24}$$

$$G_1 = -(1 - v_s^2) / E_s \tag{A.25}$$

$$H_5 = J_2^2 - J_3^2 \tag{A.26}$$

$$H_4 = R_{dc} \alpha_c r_c (T_1 + T_2 + T_3 + T_4) \tag{A.27}$$

$$H_3 = \frac{r_i r_s^2 H_2}{(1+v_s)(r_s^2 - r_i^2)} - \frac{r_i r_s^2 H_2}{(1-v_s)(r_i^2 - r_s^2)} \tag{A.28}$$

$$H_2 = \frac{1 - v_s^2}{E_s} \left\{ \frac{\alpha_s E_s R_{dc} \rho_s}{2\pi} \left[\frac{r_i^2 \ln(r_i)}{2 r_s^2} - \frac{r_i^2}{4 r_s^2} - \frac{\ln(r_s)}{2} + \frac{1}{4} \right] + \frac{\alpha_s E_s R_{dc} (r_s^2 - r_i^2)}{2 r_s^2} \left[\frac{\rho_s \ln(r_s)}{2\pi} + T_3 + T_4 \right] \right\} \tag{A.29}$$

$$H_1 = \left[\phi \alpha_{io} + (1-\phi) \alpha_{ip} \right] R_{dc} \left\{ \rho_i \left[\frac{r_c^2 \ln(r_c)}{2} - \frac{r_c^2}{4} - \frac{r_i^2 \ln(r_i)}{2} + \frac{r_i^2}{4} \right] + \pi (r_i^2 - r_c^2) \left[\frac{\rho_i \ln(r_i)}{2\pi} + T_2 + T_3 + T_4 \right] \right\} \tag{A.30}$$

$$J_5 = 2 \left[(H_3 J_2 - H_4 J_3) I^2 + J_2 L_2 P_3 + J_2 r_i - J_3 r_c - \frac{J_4}{2\pi} \right] \tag{A.31}$$

$$J_4 = -\pi (r_i^2 - r_c^2) \left[\phi \beta_{io} + (1-\phi) \beta_{ip} \right] \tag{A.32}$$

$$J_3 = -r_c (1 - v_c) / E_c \tag{A.33}$$

$$J_2 = \frac{-r_i^3 J_1}{(1+v_s)(r_s^2 - r_i^2)} + \frac{r_i r_s^2 J_1}{(1-v_s)(r_i^2 - r_s^2)} \tag{A.34}$$

$$J_1 = -(1 - v_s^2) / E_s \tag{A.35}$$

$$L_3 = \left(H_3^2 - H_4^2 \right) I^4 + 2 \left(H_3 r_i - H_4 r_c - \frac{H_1}{2\pi} \right) I^2 + 2 H_3 L_2 I^2 P_3 + 2 L_2 r_i P_3 + L_2^2 P_3^2 \tag{A.36}$$

$$L_2 = \frac{r_i r_s^2 L_1}{(1+v_s)(r_s^2 - r_i^2)} - \frac{r_i r_s^2 L_1}{(1-v_s)(r_i^2 - r_s^2)} \tag{A.37}$$

$$L_1 = -(1 - v_s^2) / E_s \tag{A.38}$$

$$M_9 = \frac{R_4 (M_3 - M_1) + (R_4 - R_1)(M_7 - M_5)}{N_2 (R_1 - R_3)} \tag{A.39}$$

$$M_8 = \frac{(Q_5 - Q_3)(M_3 - M_1) - (M_7 - M_5) Q_2}{(Q_5 - Q_3)(R_1 - R_3) - Q_2 R_4} \tag{A.40}$$

$$M_7 = \frac{-2 M_6 r_i r_s^2 (1 - v_s)}{r_i^2 - r_s^2} \tag{A.41}$$

$$M_6 = \frac{(1+v_s) \alpha_s R_{dc}}{1 - v_s} \left\{ \frac{\rho_s}{2\pi} \left[\frac{r_i^2 \ln(r_i)}{2 r_s^2} - \frac{r_i^2}{4 r_s^2} - \frac{\ln(r_s)}{2} + \frac{1}{4} \right] + \frac{r_s^2 - r_i^2}{2 r_s^2} \left[\frac{\rho_s \ln(r_s)}{2\pi} + T_3 + T_4 \right] \right\} \tag{A.42}$$

$$M_5 = M_4 + \frac{-M_2 r_i^3 (1 - 2v_i) - M_2 r_i r_c^2}{r_c^2 - r_i^2} \tag{A.43}$$

$$M_4 = \frac{(1+v_i) \alpha_i R_{dc}}{1 - v_i} \left\{ \frac{\rho_s}{2\pi} \left[\frac{r_c^2 \ln(r_c)}{2 r_i^2} - \frac{r_c^2}{4 r_i^2} - \frac{r_i \ln(r_i)}{2} + \frac{r_i}{4} \right] + \frac{r_i^2 - r_c^2}{2 r_i} \left[\frac{\rho_i \ln(r_i)}{2\pi} + T_2 + T_3 + T_4 \right] \right\} \tag{A.44}$$

$$M_3 = \frac{-2 M_2 r_c r_i^2 (1 - v_i)}{r_c^2 - r_i^2} \tag{A.45}$$

$$M_2 = \frac{(1+v_i) \alpha_i R_{dc}}{1 - v_i} \left\{ \frac{\rho_i}{2\pi} \left[\frac{r_c^2 \ln(r_c)}{2 r_i^2} - \frac{r_c^2}{4 r_i^2} - \frac{\ln(r_i)}{2} + \frac{1}{4} \right] + \frac{r_i^2 - r_c^2}{2 r_i^2} \left[\frac{\rho_i \ln(r_i)}{2\pi} + T_2 + T_3 + T_4 \right] \right\} \tag{A.46}$$

$$M_1 = R_{dc} \alpha_c r_c (1 + v_c)(T_1 + T_2 + T_3 + T_4) \tag{A.47}$$

$$N_4 = \frac{(R_3 - R_1)(Q_5 - Q_3) + Q_2 R_4}{N_2 (R_1 - R_3)} \tag{A.48}$$

$$N_3 = \frac{Q_2 N_2}{(Q_5 - Q_3)(R_1 - R_3) - Q_2 R_4} \tag{A.49}$$

$$N_2 = \frac{-2 N_1 r_i r_s^2 (1 - v_s)}{r_i^2 - r_s^2} \tag{A.50}$$

$$N_1 = -(1 + v_s) / E_s \tag{A.51}$$

$$Q_5 = \frac{Q_4 r_i^3 (1 - 2v_s) + Q_4 r_i r_s^2}{r_i^2 - r_s^2} \tag{A.52}$$

$$Q_4 = -(1 + v_s) / E_s \tag{A.53}$$

$$Q_3 = \frac{-Q_1 r_i^3 (1 - 2v_i) - Q_1 r_i r_c^2}{r_c^2 - r_i^2} \tag{A.54}$$

$$Q_2 = \frac{-2 Q_2 r_c r_i^2 (1 - v_i)}{r_c^2 - r_i^2} \tag{A.55}$$

$$Q_1 = -(1 + v_i) / E_i \tag{A.56}$$

$$R_4 = \frac{R_2 r_i r_c^2 (1 - 2v_i) + R_2 r_i r_c^2}{r_c^2 - r_i^2} \tag{A.57}$$

$$R_3 = \frac{R_2 r_c^3 (1 - 2v_i) + R_2 r_c r_i^2}{r_c^2 - r_i^2} \tag{A.58}$$

$$R_2 = -(1 + v_i) / E_i \tag{A.59}$$

$$R_1 = -(1 - 2v_c)(1 + v_c) r_c / E_i \tag{A.60}$$

$$S_4 = R_{dc} \alpha_c r_c (1 + v_c)(T_1 + T_2 + T_3 + T_4) \tag{A.61}$$

$$S_3 = \frac{-2 S_2 r_i r_s^2 (1 - v_s)}{r_i^2 - r_s^2} \tag{A.62}$$

$$S_2 = \frac{(1+v_s) \alpha_s R_{dc}}{1 - v_s} \left\{ \frac{\rho_s}{2\pi} \left[\frac{r_i^2 \ln(r_i)}{2 r_s^2} - \frac{r_i^2}{4 r_s^2} - \frac{\ln(r_s)}{2} + \frac{1}{4} \right] + \frac{r_s^2 - r_i^2}{2 r_s^2} \left[\frac{\rho_s \ln(r_s)}{2\pi} + T_3 + T_4 \right] \right\} \tag{A.63}$$

$$S_1 = \left[\phi \alpha_{io} + (1-\phi) \alpha_{ip} \right] R_{dc} \left\{ \rho_i \left[\frac{r_c^2 \ln(r_c)}{2} - \frac{r_c^2}{4} - \frac{r_i^2 \ln(r_i)}{2} + \frac{r_i^2}{4} \right] + \pi (r_i^2 - r_c^2) \left[\frac{\rho_i \ln(r_i)}{2\pi} + T_2 + T_3 + T_4 \right] \right\} \tag{A.64}$$

$$W_4 = -r_c (1 - 2v_c)(1 + v_c) / E_c \tag{A.65}$$

$$W_3 = \frac{W_2 r_i^3 (1 - 2v_s) + W_2 r_i r_s^2}{r_i^2 - r_s^2} \tag{A.66}$$

$$W_2 = -(1 + v_s) / E_s \tag{A.67}$$

$$W_1 = -\pi (r_i^2 - r_c^2) \left[\phi \beta_{io} + (1-\phi) \beta_{ip} \right] \tag{A.68}$$

$$X_2 = \frac{-2 X_1 r_i r_s^2 (1 - v_s)}{r_i^2 - r_s^2} \tag{A.69}$$

$$X_1 = -(1 + v_s) / E_s \tag{A.70}$$